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Kinetic Model of Landing Mat Performance

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Theme

PREVIOUS design criteria for portable airfield surface structures, commonly referred to as landing mat, have focused on the structural integrity of individual mat panels and the mat-soil interaction[1,2]. In 1970, however, interest in the dynamic response of mat was generated after the AM2 portion of a Tri-service mat runway failed at Dyess AFB.[3] The same landing field had been used extensively by C-130, C-141, and C-7A aircraft. Although the much heavier C-5A was supported by additional wheels to provide flotation comparable to the C-130, the increased horizontal thrust produced a bow wave and the runway buckled after only the fourth landing. Both a 1/7 scale physical model and an analytical model were developedto study the dynamic response of landing mat. This Synoptic reports the analytical model, which provides a tool to approximate the kinetics of several types of landing mat, as related to horizontal thrust, friction, laying patterns, subgrades, joint flexibility, etc.

Content

The model consists of a series of discrete rigid elements interconnected and suspended by springs and dashpots to simulate the mat-mat and mat-soil interaction, as shown in Fig. 1. Depending on the laying pattern, the model elements may represent prototype panels to the front and rear of either the total aircraft or an aircraft wheel. The degrees of freedom of each mat element include longitudinal and vertical translation, and mat rotation about a transverse axis through the mass center of the element.

The governing equations of motion are obtainable by the application of Newton's laws. Summation of vertical forces on the first mass results in the equation

$$m_1\ddot{q}_1 = -\left[KV_1\left(q_1 - \frac{L}{2}\sin q_3\right) + KV_2\left(q_1 + \frac{L}{2}\sin q_3\right) + CV_2\left(\dot{q}_1 + \frac{L}{2}\dot{q}_3\cos q_3\right)\right] + Q + \mathrm{CV}_1\left(\dot{q}_1 - \frac{L}{2}\dot{q}_3\cos q_3\right) \quad (1)$$

where $\dot{q}_1$ and $\ddot{q}_1$ are time derivatives and L the element length. Since mat elements are rigid and interconnected, vertical displacements for subsequent elements follow as

$$Y_1 = q_1 - (L/2)\sin q_3 \qquad Y_2 = q_1 + (L/2)\sin q_3 \quad (2a)$$

$$Y_n = q_1 + (L/2)\sin q_3 + \sum_{k=3}^{n} L\sin q_{2k-1} \quad (2b)$$

where Y_n is the vertical deflection of the left end of the nth mass. Summation of forces on the first mass in the longitudinal direction yields

$$m_1\ddot{q}_2 = -KL_1\dot{q}_2 - CL_1\dot{q}_2 - CL_2(\dot{q}_2 - \dot{q}_4) - KL_2(q_2 - q_4) - CF(\mathrm{sign}\,\dot{q}_2) + F + \Big[\frac{L}{2} \times [-KL_1(1-\cos q_3) - CL_1\dot{q}_3]\sin q_3 + KL_2(1-\cos q_3) + KL_2(1-\cos q_5) + CL_2(\dot{q}_3\sin q_3 + \dot{q}_5\sin q_5)]\Big] \quad (3)$$

Equating moments and rate of angular momentum on the first mass, one obtains

$$I_1\ddot{q}_3 = -CT_1\dot{q}_3 - CT_2(\dot{q}_3 - \dot{q}_5) - CL_1\dot{q}_2\frac{L}{2}\sin q_3 + CL_2(\dot{q}_2 - \dot{q}_4)\frac{L}{2}\sin q_3 + CV_1\left(\dot{q}_1 - \frac{L}{2}\dot{q}_3\cos q_3\right)\frac{L}{2}\cos q_3 - CV_2\left(\dot{q}_1 + \dot{q}_3\frac{L}{2}\cos q_3\right) \times \frac{L}{2}\cos q_3 - KT_1 q_3 - KT_2(q_3 - q_5) - KL_1 q_2\frac{L}{2}\sin q_3 + KL_2(q_2 - q_4)\frac{L}{2}\sin q_3 + KV_1\left(q_1 - \frac{L}{2}\sin q_3\right)\frac{L}{2}\cos q_3 - KV_2\left(q_1 + \frac{L}{2}\sin q_3\right)\frac{L}{2}\cos q_3 - \Big[\left(\frac{L}{2}\right)^2\sin q_3[KL_1 \times (1-\cos q_3) + CL_1\dot{q}_3\sin q_3 + KL_2(1-\cos q_3) + KL_2(1-\cos q_5) + CL_2(\dot{q}_3\sin q_3 + \dot{q}_5\sin q_5)]\Big] \quad (4)$$

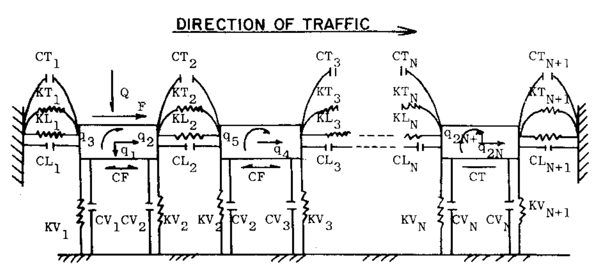

Fig. 1 Landing mat model. Variables are defined as follows: q_1 is the vertical position of the first mass; q_{2n} and q_{2n+1} are horizontal and rotational positions of the nth element, respectively; KL_n, KV_n, and KT_n are elastic spring constants, and CL_n, CV_n and CT_n are damping coefficients, where n refers to the element number; F and Q are aircraft thrust and weight; CF is the friction force and f the coefficient of friction. At the end element $n = N$.

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In a similar manner, longitudinal and rotational equations for the nth mass follow as

$$
\begin{aligned}
m_n \ddot{q}_{2n} = & -KL_n (q_{2n} - q_{2n-2}) - CL_n (\dot{q}_{2n} - \dot{q}_{2n-2}) \\
& -KL_{n+1} (q_{2n} - q_{2n+2}) - CL_{n+1} (\dot{q}_{2n} - \dot{q}_{2n+2}) \\
& - CF(\text{sign}\ \dot{q}_{2n}) \Big[-KL_n \Big(L - \frac{L}{2} \cos q_{2n-1} - \frac{L}{2} \cos q_{2n+1}\Big) \\
& - CL_n \Big(\frac{L}{2} \sin q_{2n-1} + \frac{L}{2} \dot{q}_{2n+1} \sin q_{2n+1}\Big) \\
& - KL_{n+1} \Big(-L + \frac{L}{2} \cos q_{2n+1} + \frac{L}{2} \cos q_{2n+3}\Big) \\
& + CL_{n+1} \Big(\frac{L}{2} \dot{q}_{2n+1} \sin q_{2n+1} + \frac{L}{2} \dot{q}_{2n+3} \sin q_{2n+3}\Big) \Big]
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
I_n \ddot{q}_{2n+1} = & - \frac{L}{2} \sin q_{2n+1} [KL_n (q_{2n} - q_{2n-2}) \\
& + CL_n (\dot{q}_{2n} - \dot{q}_{2n-2}) + KL_{n+1} (q_{2n+2} - q_{2n}) \\
& + CL_{n+1} (\dot{q}_{2n+2} - \dot{q}_{2n})] + \frac{L}{2} \cos q_{2n+1} (KV_n Y_n \\
& + CV_n \dot{Y}_n) - \frac{L}{2} \cos q_{2n+1} (KV_{n+1} Y_{n+1} + CV_{n+1} \dot{Y}_{n+1}) \\
& - KT_n (q_{2n+1} - q_{2n-1}) - CT_n (\dot{q}_{2n+1} - \dot{q}_{2n-1}) \\
& + KT_{n+1} (q_{2n+3} - q_{2n+1}) + CT_{n+1} (\dot{q}_{2n+3} - \dot{q}_{2n+1}) \\
& \Big[- \frac{L}{2} \sin q_{2n+1} [KL_n \Big(L - \frac{L}{2} \cos q_{2n-1} - \frac{L}{2} \cos q_{2n+1}\Big) \\
& + CL_n \Big(\frac{L}{2} \dot{q}_{2n-1} \sin q_{2n-1} + \frac{L}{2} \dot{q}_{2n+1} \sin q_{2n+1}\Big) \\
& + KL_{n+1} \Big(L - \frac{L}{2} \cos q_{2n+1} - \frac{L}{2} \cos q_{2n+3}\Big) \\
& + CL_{n+1} \Big(\frac{L}{2} \dot{q}_{2n+1} \sin q_{2n+1} + \frac{L}{2} \dot{q}_{2n+3} \sin q_{2n+3}\Big)] \Big]
\end{aligned}
\tag{6}
$$

Equations for the last mass N may be determined from Eqs. (5) and (6) by setting $q_{2n+2} = q_{2n+3} = 0$ and $n = N$.

It is noted that any moment contribution by Coulomb friction is neglected, since the mat thickness is small. Also, the equations assume continuous springs and dashpots, which implies that the transverse joints are closed initially and subsequent horizontal forces simply compress the mat more tightly. In the event of rotation, however, the prototype mat joints can partially open with no longitudinal or rotation resistance. This resistance-free joint displacement can be approximated in low profile waves by disregarding the bracketed terms in the equations. In the initial equilibrium position, the mat elements are supported by the subgrade and no appreciable vertical shear exists at the joints. When a mat element leaves the equilibrium position, however, the joints are subjected to vertical shear.

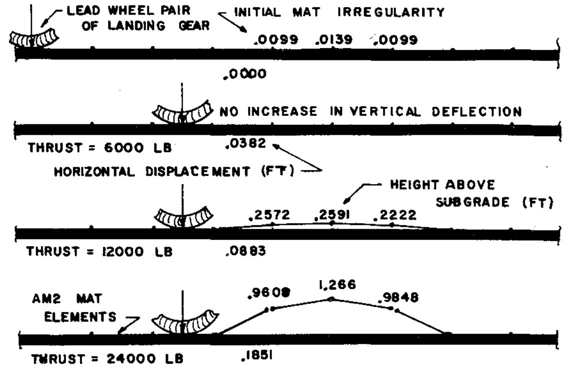

Fig. 2 Model simulation of AM2 prototype mat subjected to different loadings. Model assumes 50 elements, touchdown velocity 90 fps, free joint rotation 10 deg, $f = 0.5$, $Q = 480{,}000$ lb, $KL = 8(10)^6$ lb/ft, $CL = 50$ lb-sec/ft, $KT = 10^5$ ft-lb/rad, $CT = 1$ ft-lb-sec/rad, $KV = 1.1(10)^6$ lb/ft, and $CV = 2(10)^4$ lb-sec/ft.

Equations (2-6) represent $2N+1$ nonlinear second-order coupled differential equations. This system may be reduced to $2(2N+1)$ first-order equations and solved numerically. To determine a solution, initial displacements and velocities must be specified. The initial vertical, horizontal, and rotational displacements and velocities of the first mass, along with initial horizontal and rotational displacements and velocities of subsequent masses, must be assigned. The model may include an arbitrary number of mats and may simulate a variety of field conditions.

For a typical application, the model may be used to simulate the dynamic response of AM2 mat subjected to three different thrusts, as shown in Fig. 2. The 12,000-lb thrust simulates the loading of a C-5A wheel pair, as occurred at Dyess AFB. Since mat compression, subgrade irregularities, and other runway conditions that existed prior to the failure are not known precisely, model parameters are approximated. A more detailed discussion of the model simulation of the AM2 prototype failure and the predicted effect of other mat modifications are reported.[4] Therein, it was found that the analytical model simulated response of a 1/7-scale physical model of the AM2 mat and the C-5A landing gear.

In summary, the analytical model contributes a tool to be used in the design and maintence of expedient airfields to predict approximately the dynamic performance of mat as related to such factors as horizontal thrust, joint flexibility, friction, mat geometry, laying patterns, and subgrade conditions.

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